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Born-Infeld dynamics in uniform electric field

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Abstract: We investigate various properties of classical configurations of the Born-Infeld theory in a uniform electric field. This system is involved with dynamics of (F,Dp) bound states, which are bound states of fundamental strings and Dp-branes. The uniform electric field can be treated as a constraint on the asymptotic behavior of the fields on the brane. BPS configurations in this theory correspond to fundamental strings attached to the (F,Dp) bound state, and are found to be stable due to force balance. Fluctuations around these stable configurations are subject to appropriate Dirichlet and Neumann boundary conditions which are identical with the ones deduced from the ordinary worldsheet description of the attached string. Additionally, non-BPS solutions are studied and related physics are discussed.

Keywords: Solitons Monopoles and Instantons, D-branes.

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Contents

1. Introduction

Recent developments in string theory are greatly indebted to the discovery of the importance of D-branes [1]. Many kinds of brane configurations have been studied, partly owing to the fact that various dimensional gauge theories live on the D-branes. The low-energy effective field theory on the D-brane is the Born-Infeld theory [2, 3, 4, 5], a non-linear electrodynamics. This particular non-linear system exhibits interesting properties, and these can be well understood in terms of string theory. One of the noteworthy facts on this system is that a deformed part of the surface of the brane (parameterized by scalar fields on the brane) corresponds to other type of branes, i.e., intersecting branes can be described by gauge theories [6, 7]. Using this correspondence, new soliton solutions in gauge theories have been found [8] motivated by particular brane configurations, and on the other hand, non-perturbative quantities in string theory have been studied [6, 9, 10] using the Born-Infeld theories.

In this paper, we concentrate on the basic properties of the worldvolume Born-Infeld system in a uniform electric field.[1] A Dp-brane with the uniform electric field can be identified with a bound state of the Dp-brane and fundamental strings (called an (F,Dp) bound state) [12, 13, 14]. We consider a fundamental string ending on

[1]Related issues are found in refs. [11].

this bound state, as a generalization of refs. [6, 7]. Though the supergravity solution representing this (F,Dp) bound state has been constructed recently [15], one of the missing important ingredients is explicit supergravity solutions representing intersecting branes [16]. In the light of understanding of the AdS/CFT correspondence, the analysis in the gauge theory side is of importance.

The organization of this paper is as follows. In section 2, we study the BPS property and stability of the configuration. Then in section 3, we investigate the (Dirichlet and Neumann) boundary conditions for the attached string by means of Born-Infeld equations of motion, and see that these boundary conditions are consistent with the ones deduced from the viewpoint of string worldsheet à la Polchinski [17]. Motivated by the fact that a "throat" type of non-BPS solutions in this Born-Infeld theory is involved with the decay of brane-anti-brane system [6, 9], we analyze non-BPS configurations in the uniform electric field in section 4. The final section is devoted to discussions, in particular on the properties of these non-BPS configurations.

2. Stability of BPS configuration

First, let us see how the uniform electric field and a point charge in it are allowed as stable BPS configurations in the low-energy effective theory of a Dp-brane extending along the directions $(012\cdots p)$. The argument on the linearized version of this system given in refs. [6] and [7] tells us that half of the worldvolume supersymmetries are preserved when the fields on the brane satisfy the following BPS condition

$$F_{0\mu} = \alpha F_{9\mu}, \quad \text{with } \alpha = \pm 1\,, \tag{2.1}$$

where $F_{9\mu}$ should be understood with a scalar field $A_9 \equiv X_9$, and the index μ runs from 0 to p. This BPS eq. (2.1) is expected to be derived also from the non-linear Born-Infeld theory [18]. Under the relation (2.1), equations of motion of the Born-Infeld theory agree with the ordinary ones in the Maxwell-scalar system. As a solution,[2] it is possible to generalize the solution adopted in ref. [6] so as to include trivially a uniform electric field[3]

$$X_9 = -A_0 = -\frac{c_p}{r^{p-2}} + Ex_3\,. \tag{2.2}$$

This configuration represents a charged particle in the background electric field $\mathbf{E} = E\hat{\mathbf{x}}_3$ which is uniform on the Dp-brane.[4] As seen from eq. (2.2), the attached

[2] We choose $\alpha = -1$ and $p \geq 3$ in this paper.

[3] This background (2.2) defines an exact conformal field theory, since the derivation in ref. [19] depends not on the explicit form of the scalar potential, but only on the BPS relation (2.1).

[4] A point electric charge in a uniform background magnetic field is not a BPS configuration. A BPS configuration with a magnetic point source in a uniform magnetic field, which corresponds to a D-string ending on the D3-brane, will be discussed in section 5. An issue concerning the uniform magnetic field is found in ref. [20].

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fundamental string is not perpendicular to the Dp-brane, because of the uniform field strength (see figure 1). In other words, the Dp-brane is now tilted in order to preserve some supersymmetries.

From the viewpoint of target space supersymmetries, it is also possible to see that the configuration given by eq. (2.2) preserves a part of the supersymmetries to be stable. For simplicity, consider the case of $p=3$. Taking T-dualities in two directions along x_1 and x_2, then the tilted D-string (on which a constant field strength exists) appears. This D-string can be interpreted as a dyonic string carrying both NeveuSchwarz-NeveuSchwarz (NS-NS) and Ramond-Ramond 2-form charges [21]. Following the argument developed in ref. [22], at the spatial infinity the conserved supercharges are 1/4 of the original ones, under the existence of this dyonic string and a fundamental string perpendicular to the (123)-plane. The T-duality transformation does not change the number of preserved supersymmetries, hence the configuration of eq. (2.2) preserves 1/4 of the original target space supersymmetries (at least at the spatial infinity).

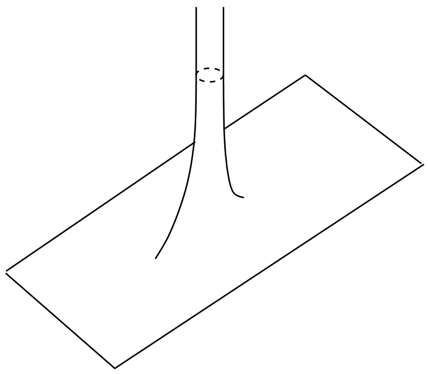

Figure 1: "Tube-like" configuration of Dp-brane surface representing a string. Now the Dp-brane is tilted, in order to be a BPS configuration, under the uniform electric field.

Although this configuration is stable because of its BPS property, naively one might be afraid of instability due to an electric force on the point charge in the uniform electric field. Actually, the charge at $r = 0$ is $(p-2)c_p\Omega_{p-1}$, and the force along the tilted direction can be read as

$$f_{\text{source}} = -(p-2)c_p\Omega_{p-1} \cdot T_p E \cdot \frac{1}{\sqrt{1+E^2}}. \quad (2.3)$$

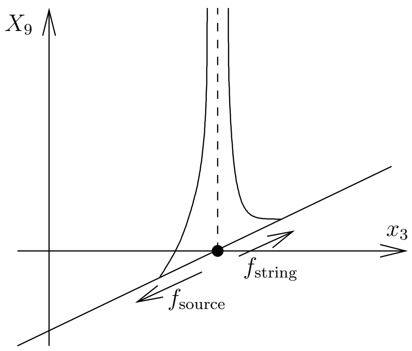

Figure 2: Force balance of the configuration.

Note that the strength of the "physical" electric field[5] is T_pE, where T_p is the tension of the Dp-brane. The final factor in eq. (2.3) is for extracting a component of the force along the tilted direction. Now, what happens is that this force (2.3) is exactly cancelled by $T_\mathrm{f} = 1/2\pi$, the tension of the fundamental string emanated from the Dp-brane (see figure 2). Actually,

[5]The charge quantization is calculated using the action

$$T_p \int d^{p+1}\sigma \sqrt{-\det(h_{\alpha\beta} + F_{\alpha\beta})},$$

therefore the Gauss law derived from this action is $\nabla \cdot (T_p\mathbf{E}) = 0$.

the contribution of the string tension along the tilted direction is

$$f_{\rm string} = \frac{E}{\sqrt{1+E^2}}\frac{1}{2\pi}\,, \tag{2.4}$$

and using the charge quantization condition $(p-2)c_p\Omega_{p-1} = g_{\rm st}(2\pi)^{p-1}$ (see ref. [6]) and $T_p = 1/(2\pi)^p g_{\rm st}$, it is easy to see the force balance as

$$f_{\rm source} + f_{\rm string} = 0\,. \tag{2.5}$$

This is consistent with the stability expected from the BPS property of the configuration.

3. Boundary conditions

3.1 Worldsheet picture

One of the remarkable properties of the description of the intersecting strings in the Born-Infeld system is that the boundary conditions for the attached strings can be reproduced [6, 23, 24]. Now, our interest is not the pure D-branes but the (F,Dp) bound state. In this case, the boundary condition in the usual worldsheet picture are known to be modified. We shall study this first, and check the consistency with the Born-Infeld theory later.

In the worldsheet approach of D-branes by Polchinski et al. [17], the boundary conditions of a fundamental string attached to the D-brane on which a uniform gauge field strength exists are [3, 5, 12]

$$\left(\partial_\sigma X^\mu - F^\mu_\nu \partial_\tau X^\nu\right)\Big|_{\sigma=0} = 0\,, \qquad (\mu = 0,\ldots,p)\,, \tag{3.1}$$

$$\partial_\tau X^i\Big|_{\sigma=0} = 0\,, \qquad (i = p+1,\ldots,9)\,. \tag{3.2}$$

For the directions transverse to the Dp-brane (Dirichlet directions), the Dirichlet boundary conditions do not change in spite of the existence of the electric field on the Dp-brane. On the other hand, for the Neumann directions, the uniform electric field $F_{03} \equiv E$ yields the so-called “mixed” boundary conditions, which relate values of two or more scalars at the boundary. How can these mixed boundary conditions be understood in terms of only a single particular scalar, say X^3 ? Defining new coordinate scalars as $X^\pm \equiv \left(X^0 \pm X^3\right)/\sqrt{2}$, then the above boundary conditions (3.1) along the plane spanned by x^0 and x^3 reads [29]

$$\left(\partial_\sigma X^\pm \pm E\partial_\tau X^\pm\right)\Big|_{\sigma=0} = 0\,. \tag{3.3}$$

A wave solution which respects this phase shift at the boundary $\sigma = 0$ is easily found as

$$X^{\pm} = A_{\pm}\left(\exp\left[i\left(\tau + \sigma \mp \frac{\alpha}{2}\right)\right] + \exp\left[i\left(\tau - \sigma \pm \frac{\alpha}{2}\right)\right]\right), \tag{3.4}$$

where the phase shift α is defined by a relation $E = \tan(\alpha/2)$, and $A_{\pm}$ is the amplitude of the wave, normalized as $|A_{\pm}| = 1$. Hence the expression for the scalar X^3 is

$$X^3 = \sqrt{2}\left(X^+ - X^-\right) = 2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)\left(\exp[i(\tau+\sigma)] + \exp\left[i(\tau-\sigma)\right]\right). \tag{3.5}$$

Here we have chosen $A_+ = -A_- = 1$, since this choice satisfies a requirement that the amplitude of the wave coming in is equal to the one of the wave going out. The expression (3.5) indicates that the boundary condition for X^3 is purely Neumann-type, not the mixed type. In the following, we will see that the above boundary conditions are reproduced in the Born-Infeld analysis.

3.2 Transverse (Dirichlet) directions

First, let us investigate the Dirichlet directions. For simplicity, we shall concentrate on the case $p = 3$ hereafter. Denoting the fluctuation in a direction transverse to both the attached string and the D3-brane as η, the equation of motion for this fluctuation is given by [6]

$$\left(1 + |\nabla X|^2\right)\ddot{\eta} - \Delta\eta = 0\,. \tag{3.6}$$

Substituting the configuration under consideration (2.2) and putting the time dependence of the fluctuation $\exp(-i\omega t)$, then eq. (3.6) becomes

$$\left[\left(1 + E^2 + \frac{2\pi E g_{\rm st}}{r^2}\cos\theta + \frac{\pi^2 g_{\rm st}^2}{r^4}\right)\omega^2 + \Delta\right]\eta = 0\,, \tag{3.7}$$

where $r \equiv \sqrt{x_1^2 + x_2^2 + x_3^2}$ and $\cos\theta \equiv x_3/r$. Now one can see that the dependence on θ appears in the equation, therefore the solution of this equation cannot be spherically symmetric. We should take into account the θ-dependence of the solution.

Assuming that the solution does not depend on another variable φ in polar coordinates, we can expand η by Legendre functions, a system of orthogonal functions, as

$$\eta(r,\theta) = \sum_{l=0}^{\infty}\eta_l(r)P_l(\cos\theta)\,. \tag{3.8}$$

We require that if one takes the limit $E \to 0$ then the solution should recover the one given in ref. [6]. It is possible to construct such a solution, and the result is (see appendix)

$$\eta(r,\theta) = \eta_0^{(0)}(r) \cdot \left(1 + \sum_{l=1}^{\infty} P_l(\cos\theta)\left[(2\pi E g_{\rm st}\omega^2)^l \prod_{i=1}^{l} \frac{1}{(i+1)(2i-1)}\right]\right) + \\ + \text{(higher order terms)}. \tag{3.9}$$

In the RHS, the "higher order terms" consist of terms which do not contribute to the phase of the total wave flux (the magnitude of those higher order waves dump as taking the limit $r \to 0$. The region $r \sim 0$ corresponds to the tip of the tube in figure 1, where the initial and final states of the wave are defined on the fundamental string.) The spherically symmetric factor $\eta_0^{(0)}(r)$ is the solution of the equation

$$\left[\left(1 + \frac{\kappa^2}{y^4}\right) + \frac{d^2}{dy^2}\right] \eta_0^{(0)}(r) = 0\,, \tag{3.10}$$

where we have put $\kappa^2 \equiv (1+E^2)\pi^2 g_{\rm st}^2 \omega^4$ and $y \equiv \pi g_{\rm st}\omega/r$. This is exactly the same one obtained in ref. [6], except for the E-dependence of κ. Using the tortoise-like coordinate

$$\xi(y) \equiv \int_{\sqrt{\kappa}}^{y} \sqrt{1+\kappa^2/y^4}\,, \tag{3.11}$$

eq. (3.10) can be rewritten as a Schrödinger-type equation

$$\left(-\frac{d^2}{d\xi^2} + V(\xi)\right)\widetilde{\eta} = \widetilde{\eta}, \quad \text{where } \widetilde{\eta} \equiv \left(1 + \frac{\kappa^2}{y^4}\right)^{1/4} \eta_0^{(0)}\,. \tag{3.12}$$

The potential $V(\xi)$ approaches to a delta-function with infinite area as one goes to the weak coupling limit $g_{\rm st} \to 0$. Thus the solution (3.9) is subject to the Dirichlet boundary condition at the weak coupling limit, as expected by the worldsheet prescription in section 3.1.

3.3 Longitudinal (Neumann) directions

For the string fluctuation in the longitudinal directions, which is described by the fluctuation of the scalar X^9, we shall follow the argument given in ref. [25]. Turning on fluctuations of both the gauge field and the scalar field X^9, the authors of ref. [25] obtained the same eq. (3.6), for the fluctuation of the gauge field, $\eta = \delta A_i$. The fluctuation of the scalar field, δX^9, is related to δA_i as

$$\partial_i A_i + \partial_t \delta X^9 = 0\,. \tag{3.13}$$

Therefore the boundary conditions for the δX^9 was found to be Neumann-type.

As discussed in ref. [25], among the various modes in δX^9, the physical mode which precisely corresponds to the fluctuation along the D-brane is the one carrying the angular momentum $l = 1$ in the worldvolume language. The solution (3.9) is composed of the modes of all l, hence it may seem to be difficult to extract only the physical mode mentioned above. In order to get the physical fluctuation along x^3, we shall turn on only δA_z. Then the relation (3.13) gives

$$-i\omega\delta X^9 \left(= \frac{\partial}{\partial z}A_z\right) = \frac{z}{r}\left(\frac{\partial}{\partial r}\eta_0^{(0)}(r) + \frac{2}{3}(\pi E g\omega^2)^2\eta_0^{(0)}(r)\right) + \cdots . \tag{3.14}$$

The second term in the parenthesis in the RHS stems from the $l = 2$ excited part in the solution (3.9), and this mode actually has Dirichlet property. However, taking the weak coupling limit, this term vanishes (and this is also the case for other terms denoted by "$\cdots$" in eq. (3.14), which indicate unphysical $l \neq 1$ modes).

The other longitudinal directions (along x^1 and x^2) can be analyzed in the same way. Summing up all together, we conclude that in the weak coupling limit, for the fluctuations along the longitudinal directions the boundary conditions are Neumann-type, as expected from the worldsheet picture.

Due to the change of the frequency parameter κ in the differential equation, there remains E-dependence at finite coupling $g_{\rm st}$. Calculating the transmitting amplitude, the total power emanated from the end of the attached string [25] is now $(1 + E^2)$ times the ordinary flux $\omega^4 g^2$. This result is natural in a sense that in the Born-Infeld system the speed of light changes under the uniform field strength background. If we turn on the background adopted in this paper in the Born-Infeld-scalar system, the velocity of the fluctuation becomes $1/\sqrt{1 + E^2}$, as seen when we neglect the terms originated from the point source in the differential eq. (3.7). This change can be viewed also as a change of the frequency ω, therefore, this results in the change of the total energy flux.

4. Analysis for non-BPS solutions

Particular non-BPS solutions of the non-linear Born-Infeld system are fascinating. The first reason is that, they are analogue of the first proposal by Born and Infeld [2] (called "BIon"or "pinched" solution). The second one is that, they are concerned with the brane- anti-brane annihilation [6, 9] (called "throat" solution, or "(charged) catenoidal" solution in ref. [7]). Now our interest is mainly on the second respect, for the case of the (F,D3) bound state.[1]

[1] The brane- anti-brane annihilation can be described also by tachyon condensation [26]. A recent result on the non-perturbative tachyon potential [27] is based on the Matrix theory, in which generally there exists a uniform gauge field strength on the constructed brane.

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We treat only static configurations. Differential equations which the scalar X ($\equiv X_9$) and the electric field $\mathbf{E}$ satisfy are as follows [6]:

$$\nabla \cdot \mathbf{\Pi} = 0\,, \tag{4.1}$$

$$\nabla \cdot \left(\frac{\nabla X + \mathbf{\Pi}(\mathbf{\Pi}\cdot\nabla X)}{\sqrt{1+|\nabla X|^2+|\mathbf{\Pi}|^2+(\mathbf{\Pi}\cdot\nabla X)^2}} \right) = 0\,. \tag{4.2}$$

Here $\mathbf{\Pi}$ is the canonical momentum associated with the gauge field $A_{1,2,3}$, defined by[2]

$$\mathbf{\Pi} = \frac{\mathbf{E}(1+|\nabla X|^2) - \nabla X(\mathbf{E}\cdot\nabla X)}{\sqrt{(1-|\mathbf{E}|^2)(1+|\nabla X|^2)+(\mathbf{E}\cdot\nabla X)^2}}\,. \tag{4.6}$$

In the BPS limit $\mathbf{E} = \nabla X$, these two eqs. (4.1) and (4.2) reduce to the ordinary Gauss law, $\nabla \cdot \mathbf{E} = 0$.

Since we want to deal with the separated parallel brane- anti-brane system, we shall fix the boundary of the brane at the spatial infinity by identifying them with the BPS case:

$$\mathbf{E} \sim \nabla X \sim E\hat{\mathbf{x}}_3 \qquad \text{at } r \sim \infty\,. \tag{4.7}$$

[2]Substituting the explicit representation of $\mathbf{\Pi}$ (eq. (4.6)) into eq. (4.2), we obtain

$$\nabla \cdot \left(\frac{\nabla X(1-|\mathbf{E}|^2) + \mathbf{E}(\mathbf{E}\cdot\nabla X)}{\sqrt{(1-|\mathbf{E}|^2)(1+|\nabla X|^2)+(\mathbf{E}\cdot\nabla X)^2}} \right) = 0\,. \tag{4.3}$$

Combined with eq. (4.1), this eq. (4.3) is invariant under the boost transformation in the plane spanned by ϕ and X [7], where ϕ is the electro-static potential, $\nabla\phi \equiv \mathbf{E}$. The throat solution in ref. [6] can be easily constructed using this invariance. Let us pursue this way of construction in our case. First, Put $\mathbf{E} = 0$ in eqs. (4.1) and (4.3). Then there remains only a single equation which should be solved:

$$\frac{\nabla X}{\sqrt{1+|\nabla X|^2}} = a\hat{\mathbf{x}}_3 + \frac{b}{r^2}\hat{\mathbf{r}}\,, \tag{4.4}$$

since eq. (4.1) is trivial due to the vanishing of the momentum $\mathbf{\Pi}$. Substituting $a = 0$ leads to the throat solution in ref. [6]. Although this method to find a solution is a clever one, we cannot use it for our purpose. This is because the solution of our interest should satisfy the boundary condition (4.7), and to attain this boundary behavior one must perform the boost on the solution (4.4) with infinite velocity. Then the boosted solution diverges and becomes meaningless.

One of the other ways to solve eqs. (4.1) and (4.3) is to put $\mathbf{E} = \hat{\mathbf{x}}_3$ (and hence $|\mathbf{E}|^2 = 1$). This assumption makes these equations easy a great deal, and the solution is

$$\nabla X = \left(c + \frac{A\cos\theta}{r^2}\right)^{-1}\left(1 + \frac{cA\cos\theta}{r^2} + \frac{q^2}{r^4}\right)\hat{\mathbf{x}}_3 + \frac{-A}{r^2}\hat{\mathbf{r}}\,, \tag{4.5}$$

where c is an integration constant. When $c \neq 1$, even with the above boost, this solution cannot be brought into the form satisfying the boundary condition (4.7). Therefore, we must put the value of c equal to 1. We find that this is precisely the special case of the general solution expressed by eq. (4.9). Putting B equal to zero in eq. (4.9), we reproduce the solution (4.5).

Substituting this boundary condition into eq. (4.6), then we have $\mathbf{\Pi} \sim E\hat{\mathbf{x}}_3$ at the spatial infinity of the worldvolume. The same argument is applied for the inside of the large parenthesis of eq. (4.2), therefore the solution of the eqs. (4.1) and (4.2) respecting the boundary conditions are written as follows:

$$\begin{aligned} \mathbf{\Pi} &= E\hat{\mathbf{x}}_3 + \frac{A}{r^2}\hat{\mathbf{r}}\,, \\ \frac{\nabla X + \mathbf{\Pi}(\mathbf{\Pi}\cdot\nabla X)}{\sqrt{1+|\nabla X|^2+|\mathbf{\Pi}|^2+(\mathbf{\Pi}\cdot\nabla X)^2}} &= E\hat{\mathbf{x}}_3 + \frac{B}{r^2}\hat{\mathbf{r}}\,. \end{aligned} \tag{4.8}$$

For simplicity, we put both A and B positive. After some manipulations we find the general solution

$$\nabla X = \frac{P\hat{\mathbf{x}}_3 + Q\hat{\mathbf{r}}}{\sqrt{R}}\,, \tag{4.9}$$

where three functions P, Q and R are assumed to depend only on r and θ in a polar coordinate system ($\cos\theta = x_3/r$), and their explicit forms are

$$P(r,\theta) \equiv E\left(1 + \frac{(A-B)E\cos\theta}{r^2} + \frac{A(A-B)}{r^4}\right), \tag{4.10}$$

$$Q(r,\theta) \equiv \frac{1}{r^2}\left[\left\{(1+E^2)B - E^2A\right\} - \frac{EA(A-B)\cos\theta}{r^2}\right], \tag{4.11}$$

$$\begin{aligned} R(r,\theta) \equiv\ & 1 + \frac{2E(A-B)\cos\theta}{r^2} \\ & + \frac{(A-B)\left\{(1-E^2)A + (1+E^2)B + E^2(A-B)\cos^2\theta\right\}}{r^4}. \end{aligned} \tag{4.12}$$

As is easily checked, at the spatial infinity $r \to \infty$, these three functions have asymptotic behavior $R \to 1$, $Q \to 0$, $P \to E$, and hence the boundary condition (4.7) is satisfied.

As expected from the case with no uniform electric field [6, 9], the BPS limit corresponds to the relation $A = B$. Actually, in this limit, three functions are simplified as $P = E$, $Q = A/r^2$ and $R = 1$, then we reproduce the BPS solution (2.2).

We can characterize the configuration of the brane by the location where ∇X diverges. This is defined by the root of the denominator of the solution, $\sqrt{R}$,

$$r^4R = \left(r^2 - E(B-A)\cos\theta\right)^2 - (B-A)\left\{(A+B) + E^2(B-A)\right\} = 0\,. \tag{4.13}$$

If $B > A$, there is a solution of this equation:

$$(r_{\text{critical}})^2 = \sqrt{(B-A)\left\{(A+B) + E^2(B-A)\right\}} + E(B-A)\cos\theta\,. \tag{4.14}$$

At this radius r_{critical}, the gradient ∇X diverges and the brane forms a "throat". Inside the critical surface defined by eq. (4.14), $r < r_{\text{critical}}(\theta)$, the solution is not defined. Thus the brane has a hole. This critical surface has a shape of stretched sphere, due to the existence of the uniform electric field. On the other hand, if $A > B$, the solution is a generalization of "BIon".[3]

The study in [6] tells us that there must be another solution which, together with the solution (4.9), forms a single throat connected smoothly. This would be an anti-brane with a hole. Now as seen above, the shape of the critical surface is distorted, therefore a naive replacement $(A, B) \to (-A, -B)$ does not lead to another solution to be joined with the brane accompanying the hole defined by eq. (4.14). Reluctantly, let us approximate the distance Δ between the brane and the anti-brane by the doubled height of the solution (4.9) estimated at $\cos\theta = 0$. Performing integration along the line defined by $\theta = \pi/2$, we have

$$\frac{\Delta}{2} \sim \frac{1}{\sqrt{1+E^2}} \cdot \frac{(1+E^2)B - E^2 A}{r_{\text{critical}}(\theta = \pi/2)} \cdot \int_1^\infty \frac{du}{\sqrt{u^4 - 1}} . \tag{4.15}$$

The limit of large separation $\Delta \longrightarrow \infty$ can be attained by the following two different limits of the parameters A and B:

$$(B - A) \longrightarrow 0 , \qquad (A + B) \text{ fixed} , \tag{4.16}$$

$$(B - A) \text{ fixed} , \qquad (A + B) \longrightarrow \infty , \tag{4.17}$$

where we suppose $(A+B)^3, (A+B) \gg (B-A)$ when taking these limits. In the limit (4.16), the critical radius of the solution becomes narrow and approaches to the BPS configuration (2.2). This solution would correspond to a fundamental string connecting the brane and the anti-brane. The limit (4.17) is the sphareron solution, discussed in detail in refs. [6, 9].

Although the general solution (4.9) satisfies various properties which will play a central role in evaluating the annihilation as seen above, unfortunately this solution has one disappointing property: there is no configuration X which yields the gradient (4.9)! In fact, one can check that rotation of the solution (4.9) does not vanish: $\nabla \times (\nabla X) \neq 0$. It follows that

$$\nabla \times (\nabla X) \; \propto \; E(B - A) . \tag{4.18}$$

At a glance, the brane seems to have thickness. At the spatial infinity, the rotation vanishes and the solution is appropriately defined, however at finite r we cannot say about the location of the brane surface. The meaning of the relation (4.18) is investigated in the last section.

[3]In the case $E^2 > 1$, there exists the "fourth" phase, in addition to the above three situations (throat, BPS and BIon). This phase appears when $0 < B < ((E^2-1)/(E^2+1))A$, and the critical surface is defined only with a restricted value of θ. This shape remind us of the non-BPS 5-branes considered in ref. [28]. However, the large uniform electric field requires careful treatment (see ref. [29]).

5. Discussions

In summary, we have investigated Born-Infeld dynamics in the uniform electric field, for both BPS and non-BPS configurations. For the BPS configuration, naive force balance ensures the stability of the configuration, and the fluctuations around the configuration satisfy precisely the boundary conditions expected from the worldsheet analysis à la Polchinski. For the non-BPS configuration, we have solved explicitly and generally the equations of motion with appropriate boundary conditions at the spatial infinity of the brane. The obtained solution is the generalization of the throat solution given in refs. [6, 7].

As mentioned in the last paragraph of section 4, the non-BPS solution presented there, eq. (4.9), is not rotation-free. This is actually a problem, since if we want to evaluate the brane- anti-brane annihilation in terms of the Born-Infeld theory, the path-integral should be performed over X and not over ∇X, hence the solution (4.9) is not relevant in the path-integral procedure. However, the solution (4.9) seems to be general, if we put the appropriate boundary condition (4.7).[4]

This unsatisfactory result may be interpreted in the following way. Equation (4.18) indicates that the vanishing of the rotation of ∇X, which is a necessary condition for the integrability, occurs when the solution is BPS ($A = B$), or, when there is no uniform electric field ($E = 0$). As for the BPS limit, it is known that the BPS solution of the ordinary Maxwell-scalar system is actually the solution of fully (higher derivative) corrected effective equations of motion of open strings [19]. BPS configurations possess this sort of generality, hence exhibit fine properties even in the uniform electric field in the approximation level of the Born-Infeld theory, as seen in section 3.

Now, how about the existence of the uniform electric field? Let us consider the case of a uniform magnetic field instead of the electric field. As shown in ref. [7], if we turn on the magnetic field, the equations of motion (and constraint equations) are manifestly invariant under the $\mathrm{SO}(2,1)$ rotation in the plane spanned by (ϕ, χ, X), where χ is the magneto-static potential. Hence the solution (4.9) is also the solution if $\mathbf{E}$ is replaced by the magnetic field $\mathbf{B}$ (with appropriately quantized magnetic charge A). For the uniform magnetic field, it is known that the low-energy effective action for the open strings is *not* the ordinary super Yang-Mills, but the one with Moyal brackets [30]. Therefore, this non-commutativity of the Moyal bracket may change the non-BPS solution to be integrable.

Finally, we comment on the relation to the supergravity calculation. In refs. [23, 24], a test string in the background of supergravity solution of D3-branes are analyzed using the string σ model approach. It was found that fluctuations of the scalar fields

[4]The other solutions (4.4) and (4.5) which do not satisfy the boundary conditions are also rotation non-free. Hence general non-BPS solutions ∇X with $E \neq 0$ seem to exhibit the non-integrability.

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on the worldsheet feels the same form of the potential as in the Born-Infeld analysis. In our case of the D3-brane with the uniform electric field, this property would be able to be confirmed in a similar manner. Recently, the supergravity solution representing the (F,Dp) bound state was constructed [14, 15]. The worldsheet action of the static probe string (which corresponds to the tube part in figure 1) in this background is[5]

$$S = \int d\tau d\sigma \left[(H')^{1/4} H^{-3/4} \sqrt{(X_3')^2 + H(X_\perp')^2} - X_3' \frac{E}{\sqrt{1+E^2}} \frac{1}{H} \right], \tag{5.1}$$

where H and H' are harmonic functions in the transverse space.[6] The second term in eq. (5.1) stems from the coupling of the probe string to the non-trivial NS-NS 2-from background produced by the fundamental strings condensed in the D3-brane. Near the infinity of the transverse space, two harmonic functions are approximated as $H \sim H' \sim 1$, hence the static solution lead from the equations of motion is $X_3' = kX_\perp'$, where k is an integration constant. Substituting this relation into the action (5.1), we have

$$S \sim \int d\tau d\sigma X_\perp' \left(\sqrt{1+k^2} - k \frac{E}{\sqrt{1+E^2}} \right) = \int d\tau X_\perp \Big|_{\text{infinity}} \left(\sqrt{1+k^2} - k \frac{E}{\sqrt{1+E^2}} \right). \tag{5.3}$$

Noting that $X_\perp$ is positive, the action is minimized at the value $k = E$. Thus at the spatial infinity we obtain the tilted configuration of the string: $X_3' = EX_\perp'$, which is exactly matched to the configuration considered in section 2.

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[5] We turn on only the relevant two scalars, X_3 (parameterizing the direction along the electric flux) and $X_\perp$ (the radial coordinate in the transverse directions).

[6] The explicit form of these functions is

$$H \equiv 1 + \frac{Q}{(X_\perp)^4}, \qquad H' \equiv 1 + \frac{Q}{(1+E^2)(X_\perp)^4}. \tag{5.2}$$

These functions give two distinct "core"(this core is defined as the value of $X_\perp$, where the first term in the harmonic function (in this case, 1) is comparable to the second term ($\sim 1/X_\perp^4$)). For this reason, the potential which the scalar feels, computed in the supergravity formulation, might be different from the one obtained in sections 3.2 and 3.3. However, in the weak coupling limit, two cores may overlap with each other and this potential will reproduce the appropriate boundary conditions considered in this paper.

A. Solution for fluctuation

In this appendix, we present a way to solve the eq. (3.7) using the expansion (3.8). Making an abbreviation

$$\mathcal{O} \equiv \left(1 + E^2 + \frac{\pi^2 g_{\rm st}^2}{r^4}\right)\omega^2 + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right), \tag{A.1}$$

the eq. (3.7) is decomposed due to the orthogonality of the Legendre functions as follows:

$$\begin{aligned} \mathcal{O}\eta_0 + \frac{2\epsilon}{r^2}\,\eta_1 &= 0\,, \\ \left(\mathcal{O} - \frac{2}{r^2}\right)\eta_1 + \frac{2\epsilon}{r^2}\left(\eta_0 + \frac{2}{5}\,\eta_2\right) &= 0\,, \\ \left(\mathcal{O} - \frac{6}{r^2}\right)\eta_2 + \frac{2\epsilon}{r^2}\left(\frac{2}{3}\,\eta_1 + \frac{3}{7}\,\eta_3\right) &= 0\,, \quad \ldots\,, \end{aligned} \tag{A.2}$$

where we put $\epsilon \equiv \pi E g_{\rm st}\omega^2$. Due to the structure of these equations, it is possible to expand η_l further as

$$\eta_l = \epsilon^l \eta_l^{(l)} + \epsilon^{l+2}\eta_l^{(l+2)} + \cdots\,, \tag{A.3}$$

where, in particular, $\eta_0^{(0)}$ is the zero mode of the operator $\mathcal{O}$. Then easily one can deduce that the leading terms of η_l in each angular momentum l satisfy relations

$$\eta_1^{(1)} = \eta_0^{(0)}, \qquad \eta_2^{(2)} = \frac{2}{9}\,\eta_1^{(1)}, \qquad \ldots\,, \tag{A.4}$$

therefore intrinsically all $\eta_l^{(l)}$ are identical with $\eta_0^{(0)}$. The next-to-leading terms $\eta_l^{(l+2)}$ are determined by evaluating the next-to-leading terms in the decomposed eq. (A.2). As an example, let us consider the first one:

$$\mathcal{O}\eta_0^{(2)} + \frac{2}{r^2}\,\eta_1^{(1)} = 0\,. \tag{A.5}$$

At $r \sim 0$ ($y \sim \infty$), the region which represents the place where the effect of the end point of the attached string is not expected to appear, the operator $\mathcal{O}$ is approximated by

$$\mathcal{O} \sim \frac{1}{\pi^2 g_{\rm st}^2 \omega^2} y^4 \left(1 + \frac{d^2}{dy^2}\right). \tag{A.6}$$

Therefore, using the asymptotic ($r \sim 0$) behavior of the solution $\eta_1^{(1)} = \eta_0^{(0)} \sim \exp(\pm iy)$ at the weak coupling limit, we obtain a solution of (A.5) as

$$\eta_0^{(2)} \sim \frac{1}{y}\exp(\pm iy)\,. \tag{A.7}$$

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This does not contribute when we discuss the phase shift of the boundary, since the magnitude of this mode is dumping fast enough at $r \sim 0$. Owing to similar argument, the following relations can be derived:

$$\eta_l^{(l+2k)} \sim y^{-k} \exp(\pm iy) \quad \text{at } r \sim 0\,. \tag{A.8}$$

These modes ($k \geq 1$) are collectively represented in eq. (3.9) as "higher order terms".

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